

Imprints of Nuclear Symmetry Energy on Properties of Neutron Stars

Bao-An Li¹, Lie-Wen Chen^{1,2}, Michael Gearheart¹, Joshua Hooker¹,
Che Ming Ko³, Plamen G. Krastev^{1,4}, Wei-Kang Lin¹, William G.
Newton¹, De-Hua Wen^{1,5}, Chang Xu^{1,6} and Jun Xu³

¹Department of Physics and Astronomy, Texas A&M University-Commerce, Commerce, Texas 75429-3011, USA

²Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, China

³Cyclotron Institute and Department of Physics and Astronomy, Texas A&M University, College Station, Texas 77843-3366, USA

⁴Department of Physics, San Diego State University, 5500 Campanile Drive, San Diego, CA 92182-1233, U.S.A.

⁵Department of Physics, South China University of Technology, Guangzhou 510641, China

⁶Department of Physics, Nanjing University, Nanjing 210008, China

E-mail: Bao-An_Li@Tamu-Commerce.edu

Abstract. Significant progress has been made in recent years in constraining the density dependence of nuclear symmetry energy using terrestrial nuclear laboratory data. Around and below the nuclear matter saturation density, the experimental constraints start to merge in a relatively narrow region. At supra-saturation densities, there are, however, still large uncertainties. After summarizing the latest experimental constraints on the density dependence of nuclear symmetry energy, we highlight a few recent studies examining imprints of nuclear symmetry energy on the binding energy, energy release during hadron-quark phase transitions as well as the w -mode frequency and damping time of gravitational wave emission of neutron stars.

1. Introduction

Properties of neutron-rich matter are at the heart of many fundamental questions in both nuclear physics and astrophysics [1]. They are currently being explored experimentally by using a wide variety of advanced new facilities, such as, Facility for Rare Isotope Beams (FRIB), x-ray satellites and gravitational wave detectors. Most critical to understanding experimental observations of various phenomena using these facilities is the Equation of State (EOS) of neutron-rich nucleonic matter. The latter can be written in terms of the energy per nucleon at density ρ as

$$E(\rho, \delta) = E(\rho, \delta = 0) + E_{\text{sym}}(\rho)\delta^2 + \mathcal{O}(\delta^4) \quad (1)$$

where $\delta \equiv (\rho_n - \rho_p)/(\rho_p + \rho_n)$ is the neutron-proton asymmetry and $E_{\text{sym}}(\rho)$ is the density-dependent nuclear symmetry energy. The latter is a vital ingredient in the theoretical description of neutron stars where the proton fraction is uniquely determined by the symmetry energy $E_{\text{sym}}(\rho)$ [2, 3], and of the structure of neutron-rich nuclei and reactions involving them [4, 5, 6, 7, 8]. It is currently the most uncertain part of the EOS of neutron-rich nuclear

matter, especially at super-saturation densities. Very recently, attempts have been made to extract meaningful constraints on the neutron star EOS, facilitated by neutron star observational data with sufficient accuracy, and from a wide enough range of neutron star phenomena associated with different density regions within the star [9, 10, 11]. This signals a new era in which the quality and quantity of astronomical data make constraining the microphysical properties of neutron star matter from sub-nuclear to super-nuclear densities a realistic endeavor. At the same time, the nuclear matter EOS is being probed with unprecedented accuracy by a variety of terrestrial nuclear experiments, see, e.g., refs. [8, 12, 13, 14, 15, 16, 17]. Combining the observational and experimental data, together with the astrophysical and nuclear modeling required to interpret them, has the potential to impose stringent constraints on the nuclear symmetry energy at both sub- and super-saturation densities, see, e.g., refs. [18, 19, 20, 21, 22, 23].

2. The key physics behind nuclear symmetry energy

Why is the nuclear symmetry energy so uncertain? To help understand this question, it is useful to first recall the relationship between the symmetry energy and the isospin-dependence of strong interaction at the mean-field level. According to the well-known Lane potential [24], the neutron/proton single-particle potential $U_{n/p}(\rho, k, \delta)$ can be well approximated by $U_{n/p}(\rho, k, \delta) \approx U_0(\rho, k) \pm U_{\text{sym}}(\rho, k)\delta$ where the $U_0(\rho, k)$ and $U_{\text{sym}}(\rho, k)$ are, respectively, the isoscalar and isovector (symmetry) potentials for nucleons with momentum k in nuclear matter of isospin asymmetry δ at density ρ . It was shown first by Brueckner, Dabrowski and Haensel [25, 26] using K-matrices within the Brueckner theory in the 60's-70's, and more recently by Xu et al. [27] using the Hugenholtz-Van Hove (HVH) theorem [28] that the nuclear symmetry energy can be explicitly expressed as

$$E_{\text{sym}}(\rho) = \frac{1}{6} \frac{\partial(t + U_0)}{\partial k} \big|_{k_F} \cdot k_F + \frac{1}{2} U_{\text{sym}}(\rho, k_F), \quad (2)$$

where $t(k) = \hbar^2 k^2 / 2m$ is the kinetic energy and $k_F = (3\pi^2 \rho / 2)^{1/3}$ is the nucleon Fermi momentum in symmetric nuclear matter at density ρ . Moreover, it was shown recently using the HVH theorem that the slope of nuclear symmetry energy at an arbitrary density ρ can be written as [27, 29]

$$L(\rho) \equiv 3\rho \frac{\partial E_{\text{sym}}(\rho)}{\partial \rho} \big|_{\rho} = \frac{1}{6} \frac{\partial(t + U_0)}{\partial k} \big|_{k_F} \cdot k_F + \frac{1}{6} \frac{\partial^2(t + U_0)}{\partial k^2} \big|_{k_F} \cdot k_F^2 + \frac{3}{2} U_{\text{sym}}(\rho, k_F) + \frac{\partial U_{\text{sym}}}{\partial k} \big|_{k_F} \cdot k_F. \quad (3)$$

The $E_{\text{sym}}(\rho)$ and $L(\rho)$ are correlated by the same underlying interaction. The above expressions for $E_{\text{sym}}(\rho)$ and $L(\rho)$ in terms of the isoscalar and isovector single-particle potentials are particularly useful for extracting the symmetry energy and its density slope from terrestrial nuclear laboratory experiments. On one hand, in many reaction models, such as transport model simulations of nuclear reactions, the EOS only enters indirectly into the reaction dynamics and affects the final observables through the single-nucleon potential $U_{n/p}(\rho, k, \delta)$. Except in situations where statistical equilibrium is established and thus many observables are directly related to the energy $E(\rho, \delta)$ after correcting for finite-size effects, what is being directly probed in nuclear reactions is the single-nucleon potential $U_{n/p}(\rho, k, \delta)$. On the other hand, in many structure models, such as shell models, the direct input is also the single nucleon potential. Thus, the isospin-momentum-density dependence of the single nucleon potential $U_{n/p}(\rho, k, \delta)$ is the key in determining the EOS of neutron-rich nuclear matter. Comparing model calculations with experimental data allows the determination of the single nucleon potential which then determines directly the $E_{\text{sym}}(\rho)$ and $L(\rho)$. While the density and momentum dependence of the isoscalar potential $U_0(\rho, k)$ has been relatively well determined up to about 4 to 5 times

the normal nuclear matter density ρ_0 using nucleon global optical potentials from nucleon-nucleus scatterings as well as kaon production and nuclear collective flow in relativistic heavy-ion collisions [12, 13], our current knowledge about the symmetry potential $U_{\text{sym}}(\rho, k)$ is rather poor especially at high density and/or momenta [8, 30, 31, 32, 33, 34]. In fact, while some models predict decreasing symmetry potentials albeit at different rates, some others predict instead increasing ones with growing nucleon momentum. Thus, the high density/momentum behavior of the symmetry potential $U_{\text{sym}}(\rho, k)$ is still very uncertain. Experimentally, there is some constraints on the symmetry potential only at normal density for low energy nucleons up to about 100 MeV obtained from nucleon-nucleus and (p, n) charge exchange reactions [27].

Of course, besides the uncertain density and momentum dependence of the symmetry potential, different approaches used in treating many-body problems in the various models also contribute to the divergence of the predicted symmetry energy especially at supra-saturation densities. Nonetheless, there are several key and commonly used physics ingredients that can affect the predicted $E_{\text{sym}}(\rho)$ in all theories. In the interacting Fermi gas model, the isovector potential at k_F can be written as [35, 36]

$$U_{\text{sym}}(k_F, \rho) = \frac{1}{4}\rho \int [V_{T1}(r_{ij})f^{T1}(r_{ij}) - V_{T0}(r_{ij})f^{T0}(r_{ij})]d^3r_{ij} \quad (4)$$

in terms of the isosinglet ($T=0$) and isotriplet ($T=1$) nucleon-nucleon (NN) interactions $V_{T0}(r_{ij})$ and $V_{T1}(r_{ij})$, and the corresponding NN correlation functions $f^{T0}(r_{ij})$ and $f^{T1}(r_{ij})$, respectively. Needless to say, if there is no isospin dependence in both the NN interaction and correlation function, then the isovector potential $U_{\text{sym}}(k_F, \rho)$ vanishes. The $E_{\text{sym}}(\rho)$ thus reflects the competition between the NN interaction strengths and correlation functions of the isosinglet and isotriplet channels. However, effects of several key ingredients, such as the tensor force, isospin-dependence of short-range nucleon-nucleon correlations and spin-isospin dependent three-body force, are not yet well understood, see, e.g., refs. [36, 37, 38, 39]. Consequently, the predicted symmetry energies diverge rather widely, especially at supra-saturation densities.

3. Current status of experimental constraints on nuclear symmetry energy

While significant progress has been made in constraining the $E_{\text{sym}}(\rho)$ as a result of the great efforts made by many people in both the nuclear physics and the astrophysics community, experimental constraints obtained so far are still not tight enough to limit stringently model predictions even around the saturation density ρ_0 . Moreover, it has been broadly recognized that significant model dependence exists for most of the observables studied so far. For instance, near ρ_0 the symmetry energy can be characterized by using the value of $E_{\text{sym}}(\rho_0)$ and the slope parameter $L(\rho_0)$. The $E_{\text{sym}}(\rho_0)$ is known to be around 28 – 34 MeV mainly from analyzing nuclear masses within liquid-drop models [40, 41, 42]. The exact value of $E_{\text{sym}}(\rho_0)$ extracted this way depends largely on the size and accuracy of nuclear mass data base as well as whether/what surface symmetry energy is used in the analysis. The latest study by Liu et al. [43] by analyzing the atomic masses indicates that the value of $E_{\text{sym}}(\rho_0)$ is between 29.4 and 32.8 MeV while the value of $L(\rho_0)$ is between 52.9 and 78.7 MeV. The density slope $L(\rho_0)$ has also been extensively studied but remains much more uncertain. Shown in the left window of Fig. 1 are the $L(\rho_0)$ values extracted from studying several phenomena/observables in nuclear structures and reactions [27, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55]. The values of $L(\rho_0)$ from these studies scatter between about 20 to 115 MeV while each individual study may have given a smaller uncertain range. Although it is very encouraging to note that the constraints seem to start converging around $L(\rho_0) \approx 60$ MeV and $E_{\text{sym}}(\rho_0) \approx 31$ MeV, it is obviously important to further narrow them down with new measurements and model analyses. It is thus encouraging to see the most recent determination of $E_{\text{sym}}(\rho_0) = 30.5 \pm 3$ MeV and $L(\rho_0) = 52.5 \pm 20$ MeV [56] from combining the constraints on $L(\rho_0)$ and $E_{\text{sym}}(\rho_0)$ based on

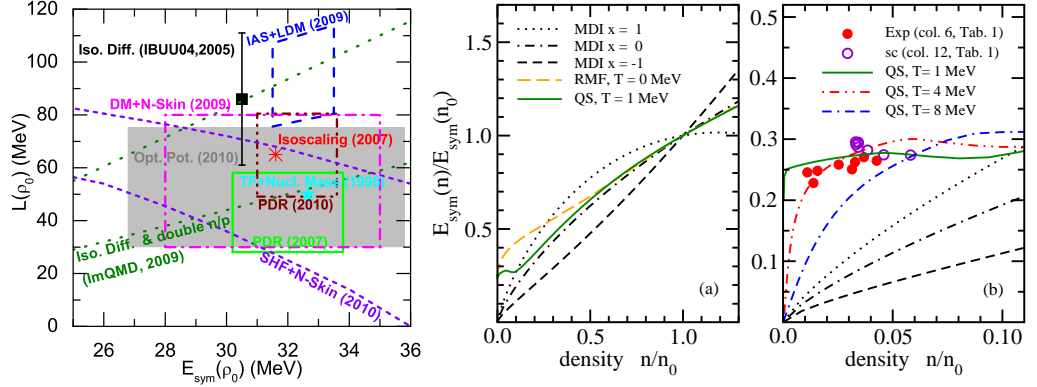


Figure 1. Left Window: Density slope versus the magnitude of the symmetry energy at normal density extracted from isospin diffusion and neutron/proton ratio of pre-equilibrium nucleon emissions within the Improved Molecular Dynamics (ImQMD-2009) model [44, 45, 46], isospin diffusion within the Isospin-Dependent Boltzmann-Uehling-Uhlenbeck (IBUU04- 2005) [47, 48], isoscaling [49], energy shift of isobaric analogue states within liquid drop model (IAS+LDM-2009) [50], neutron-skins of several heavy nuclei using the droplet model (DM) or the Skyrme-Hartree-Fock (SHF) approach [51, 52, 53], pygmy dipole resonances (PDR) in ^{209}Pb , ^{68}Ni and ^{132}Sn [54, 55], and the nucleon global optical potentials (GOP) [27]. Taken from ref. [27]. **Right Window:** Comparisons of the scaled symmetry energy as a function of the scaled total density n/n_0 for different approaches and the experiment. Left panel: Symmetry energies from the MDI parametrization of Chen et al. [47, 65] for $T = 0$ and different $L(\rho_0)$, controlled by the parameter x (dotted, dot-dashed and dashed (black) lines); from the quantum-statistical (QS) approach including light clusters for temperature $T = 1$ MeV (solid (green) line), and from the relativistic mean-field (RMF) model at $T = 0$ including heavy clusters (long-dashed (orange) line). Right panel: The internal scaled symmetry energy in an expanded low-density region. Shown are again the MDI curves and the QS results for $T = 1, 4$, and 8 MeV compared to the experimental data using the entropy from the nuclear statistical equilibrium (NSE) model (solid circles) and the results of the self-consistent calculation (open circles). Taken from ref. [63].

recently extracted $E_{\text{sym}}(\rho_A = 0.1 \text{ fm}^{-3})$ [43, 58, 59] with those from recent analyses [53] of neutron skin thickness of Sn isotopes in the same SHF approach. At very low densities, the formation of clusters is energetically favored. The symmetry energy of the clustered matter is expected to be different from that of uniform matter, see, e.g., refs. [60, 61, 62]. Indeed, it was recently found in experiments by Natowitz et al. that the symmetry energy of clustered matter remains finite as the average density approaches zero [63]. As shown in the right window of Fig. 1, conventional theoretical calculations of the symmetry energy of uniform matter based on mean-field approaches (e.g., the MDI interaction[64, 47, 65]) fail to give the correct low-temperature, low-density limit that is governed by correlations, in particular by the appearance of bound states. A recently developed quantum statistical (QS) approach [62] that takes the formation of clusters into account predicts symmetry energies that are in very good agreement with the experimental data. At supra-saturation densities, the situation is much worse. In fact, even the trend of the symmetry energy, namely, whether it increases or decreases with increasing density, is still controversial partially because of the very limited data available and the less known clean probes compared to the situation at sub-saturation densities. Among all observables studied so far, the neutron-proton differential flow [66], π^-/π^+ ratio [67] and the squeeze-out of the neutron/proton ratio [68] especially at high transverse momenta in heavy-ion reactions are among the most promising probes of the high density behavior of the nuclear symmetry energy. Unfortunately, the conclusions are still model dependent [69, 70, 71, 72, 73].

4. Impacts of nuclear symmetry energy on properties of neutron stars

It is well known that many critical issues in astrophysics, such as the composition and the cooling rate of proto-neutron stars as well as the core-crust transition density, properties of the pasta phase, the mass-radius correlation and the moment of inertia of neutron stars, all depend sensitively on the $E_{\text{sym}}(\rho)$. Many research papers and review articles have been written on these topics, see, e.g., refs. [2, 3, 74]. Most of these studies have concentrated on astrophysical effects of the symmetry energy at sub-saturation densities. However, the nuclear symmetry energy at supra-saturation densities has long been considered by some astrophysicists as the most uncertain one among all properties of dense neutron-rich nucleonic matter [75, 76, 77]. More efforts are thus need to explore many possible astrophysical effects of the symmetry energy at supra-saturation densities [78]. In this section, as examples we highlight a few recent studies on effects of the symmetry energy, especially its high-density behavior, on the binding energy of neutron stars, total energy release during mini-collapse triggered by hadron-quark phase transitions inside neutron stars, and the w -mode of gravitational waves from neutron stars.

4.1. Effects of symmetry energy on the binding energy of neutron stars

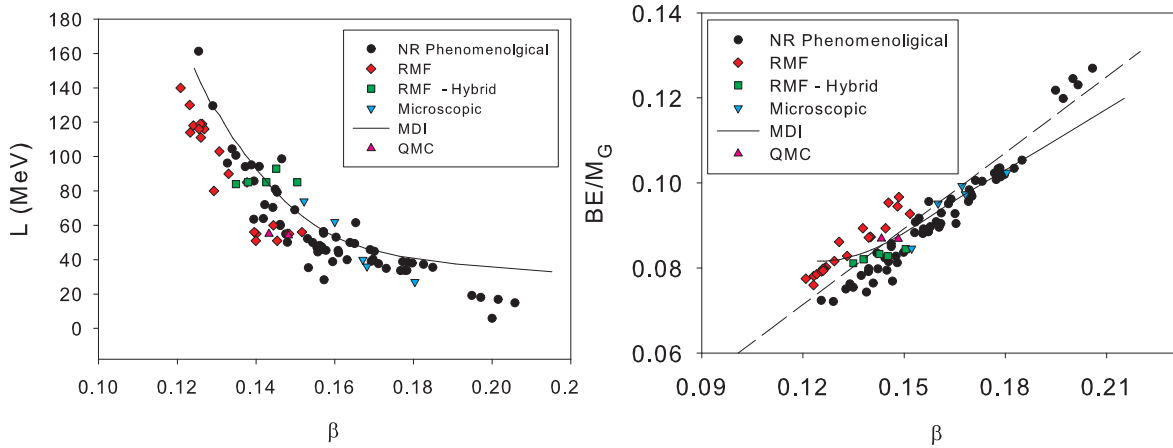


Figure 2. Left Window: Correlation between the slope parameter $L(\rho_0)$ of the symmetry energy and the compactness parameter $\beta = GM_G/Rc^2$ for the sample of EoSs. **Right Window:** The fractional gravitational binding energy of the neutron star BE/M_G versus the compactness parameter β for the sample of EoSs. The dashed line indicates the analytic Newtonian result. Taken from ref. [23].

The binding energy of a neutron star is given by [79] $BE = M_G - M_{\text{bar}}$, where M_G is the gravitational mass of the neutron star measured from infinity [80], i.e.,

$$M_G = \int_0^R 4\pi r^2 \epsilon(r) dr \quad (5)$$

with $\epsilon(r)$ being the energy density at radius r . The M_{bar} is the baryonic mass of the neutron star, namely, the mass when all the matter in the neutron star is dispersed to infinity [79]. It can be calculated from $M_{\text{bar}} = NM_B$, where $M_B = 1.66 \times 10^{-24}g$ is the nucleon mass and N is the total number of baryons [79], i.e.,

$$N = \int_0^R 4\pi r^2 \left[1 - \frac{2m(r)}{r} \right]^{-1/2} \rho_B(r) dr \quad (6)$$

with $\rho_B(r)$ being the baryon density profile of the neutron star. For a given EOS, the binding energy can be readily obtained by first solving the Tolman-Oppenheimer-Volkov (TOV) equation. Newton et al. [23] examined the correlation among the symmetry energy slope $L(\rho_0)$, pressure at normal density $P(n_0)$, compactness of neutron stars $\beta = GM_G/Rc^2$ and the binding energy. As shown in Figs. 2, for a wide range of EoSs, the binding energy correlates roughly linearly with the compactness of the neutron star β , which in turn correlates strongly with the slope of the nuclear symmetry energy $L(\rho_0)$ predicted by the EoSs. Therefore one expects to see a correlation between $L(\rho_0)$ and the binding energy of the neutron star, and consequently the baryon mass, given that the gravitational mass is fixed. Thus, laboratory constraints on L can be used to extract the constraint on the baryon mass of neutron stars. This approach was used to constrain the lower mass member of the double pulsar binary system, PSR J0737-3039B. Comparing with independent constraints derived from modeling the progenitor star of J0737-3039B up to and through its collapse under the assumption that it was formed in an electron capture supernova, it was found that the two sets of constraints are consistent only if $L(\rho_0) \leq 70$ MeV [23].

4.2. Effects of symmetry energy on the energy release during hadron-quark phase transition in neutron stars

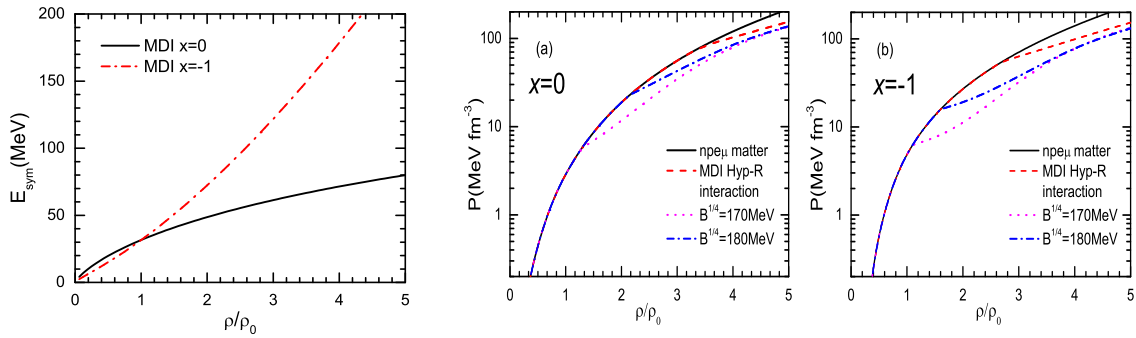


Figure 3. **Left Window:** Density dependence of nuclear symmetry energy from the MDI interaction with parameter $x = 0$ and -1 . **Right Windows:** EOSs of pure $npe\mu$ matter (nucleonic), hyperonic matter (MDI Hyp-R interaction) and hybrid stars for the bag constant $B^{1/4} = 180$ MeV and 170 MeV with $x = 0$ (a) and $x = -1$ (b).

The total energy release E_g during the hadron-quark phase transition is the difference of binding energy before and after the micro-collapse. It reduces to the difference in the gravitational masses for the hadronic (h) and hybrid (q) configurations of the neutron star as a result of baryon number conservation, namely, $E_g = M_{G,h} - M_{G,q}$. Thus, the E_g should depend on the EOSs of dense matter either in the hadronic or quark phase. For the neutron-rich hadronic matter, the most uncertain part of its EOS is the density dependence of the nuclear symmetry energy, while for the quark matter within the MIT bag model, the most uncertain part of its EOS is the bag constant. The relative effects of the density dependence of the nuclear symmetry energy and the bag constant on the total energy release due to the hadron-quark phase transition in NSs were recently examined by Lin et al. [81]. Shown in Fig. 3 are several model EOSs for hybrid stars obtained from an isospin- and momentum-dependent effective interaction (MDI) [64, 82] for the baryon octet, the MIT bag model for the quark matter [83, 84] and the Gibbs construction for the hadron-quark phase transition [85, 86]. For comparisons, the pure $npe\mu$ (labeled as nucleonic) and hyperonic (labeled as MDI Hyp-R interaction) EOSs are also included. As it is well known, the appearance of hyperons and the hadron-quark phase transition softens the EOS of neutron star matter. The EOSs with $x = 0$ and $x = -1$ are about the same

below and around the saturation density. However, it is interesting to see that the starting point and the degree of softening due to the appearance of hyperons are sensitive to the $E_{\text{sym}}(\rho)$ at high densities. Moreover, the $E_{\text{sym}}(\rho)$ also affects appreciably the starting point of the hadron-quark mixed phase, especially with the larger bag constant. Nevertheless, it is obvious that the starting point is much more sensitive to the bag constant B for a given symmetry energy parameter x . These features are consistent with those first noticed by Kutschera et al. [87].

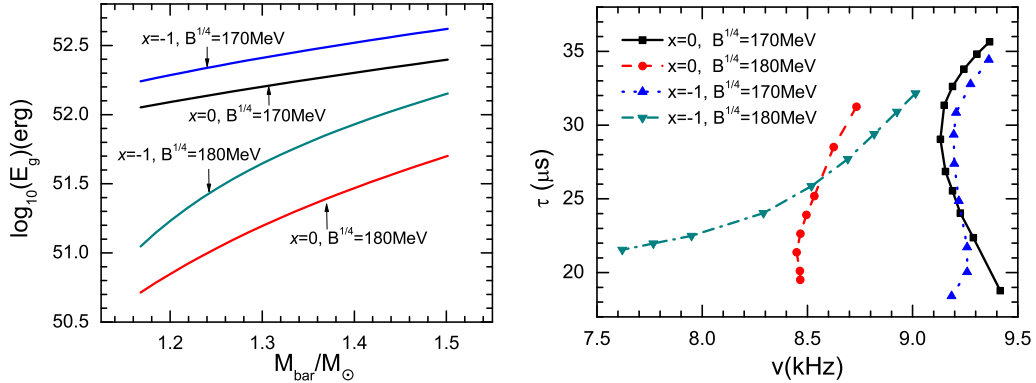


Figure 4. Left Window: Total energy release due to the hadron-quark phase transition as a function of the baryonic mass of a neutron star. **Right Window:** Damping time versus frequency of the first w -mode for hybrid stars.

Shown in the left window of Fig. 4 is the energy release as a function of the baryonic mass M_{bar}/M_{\odot} of a neutron star. It is seen that the energy release increases with M_{bar}/M_{\odot} and is higher with a smaller ($B^{1/4} = 170$ MeV) bag constant B but a stiffer ($x = -1$) symmetry energy. Effects of varying the bag constant B are obviously more significant than varying the symmetry energy parameter x , especially on the core mass and thus the energy release. The variation of the bag constant B also affects appreciably the radii of hybrid stars. On the other hand, the variation of the symmetry energy parameter x only has appreciable effects on the radii of both hyperonic and hybrid stars. Its effects on the energy release is much smaller than the bag constant B .

4.3. Imprints of symmetry energy on the w -mode of gravitation waves

The search for gravitational wave signals is another forefront in astrophysics and cosmology. Gravitational waves are tiny disturbances in space-time and are a fundamental, although not yet directly confirmed, prediction of General Relativity. They would open up an entirely new non-electromagnetic window for probing the physics that is hidden or dark to current electromagnetic observations. There are many possible sources of gravitational waves. For instance, the hadron-quark phase transition can trigger various oscillation modes of neutron stars. When coupled with rotation, these oscillations generate gravitational waves. The energy release discussed in the previous subsection determines the maximum strain amplitude of emitted gravitational waves. Moreover, the strain amplitude of gravitational waves from pulsars depends on their quadrupole deformation which, in turn, is determined by the EOS of neutron-rich nucleonic matter. Furthermore, according to Chandrasekhar & Ferrari [88], the frequency and damping time of the w -mode of gravitational waves from oscillating neutron stars are described by a unique second-order differential equation with the EOS as the only input. Indeed, it was recently shown that the strain amplitude from elliptically deformed pulsars as well as the frequency and damping time of the w -mode of oscillating neutron stars are all strongly influenced by the density

dependence of the nuclear symmetry energy [89, 90, 91, 92, 93]. Thus, an accurate determination of the nuclear symmetry energy is also important for detecting gravitational waves using various ground-based and satellite-based detectors, such as, LIGO [94] and LISA [95]. As a quantitative example, shown in the right window of Fig. 4 is the frequency versus damping time of the first axial w -mode. For the softer EOS with $B^{1/4} = 170$ MeV, the effect of the symmetry energy is small. The density dependence of the symmetry energy can, however, significantly affect the frequency of the w -mode for the stiffer EOS with $B^{1/4} = 180$ MeV as a result of the higher hadron-quark transition density. In this case, the symmetry energy in the hadronic phase then also plays an important role in determining the structure of NSs. Nevertheless, comparing the frequencies of the w -mode for the same x parameter but different values of B , it is clear that the bag constant has a much stronger effect as it changes the underlying EOS of dense matter more significantly.

5. Summary

In summary, important progress has been made in recent years in constraining the symmetry energy, especially around and below the saturation density, with terrestrial nuclear laboratory data. Nevertheless, the field is still at its beginning. While a number of potentially useful probes of the symmetry energy, especially at supra-saturation densities, have been proposed, available experimental data are mostly for reactions with stable beams. Coming experiments with more neutron-rich nuclei at several advanced high energy radioactive beam facilities are expected to improve the situation dramatically. Thus, more exciting times are yet to come. Implications of the partially constrained symmetry energy on some astrophysical phenomena have also been explored. On the other hand, impressive progress has also been made in extracting information about the EOS from astrophysical observations. Hopefully, working together and combining results from both fields will allow us to finally pin down the EOS of neutron-rich nuclear matter at both sub- and super-saturation densities in the near future.

6. Acknowledgments

This work was supported in part by the US National Science Foundation Grant Nos. PHY-0757839 and PHY-0758115, the National Aeronautics and Space Administration under grant NNX11AC41G issued through the Science Mission Directorate, the Welch Foundation under Grant No. A-1358, the Texas Coordinating Board of Higher Education Grant No.003565-0004-2007, the Young Teachers' Training Program from China Scholarship Council under Grant No. 2007109651, the National Natural Science Foundation of China under Grant No.10947023, 10735010, 10775068, 10805026 and 10975097, and the Fundamental Research Funds for the Central University, China under Grant No.2009ZM0193, Shanghai Rising-Star Program under grant No. 11QH1401100, the National Basic Research Program of China (2007CB815004 and 2010CB833000).

References

- [1] C.J. Horowitz, arXiv:1008.0402, plenary talk at INPC2010, in this volume.
- [2] J.M. Lattimer and M. Prakash, Phys. Rep. **333-334**, 121 (2000); Astrophys. J. **550**, 426 (2001); Science **304**, 536 (2004); Phys. Rep. **442**, 109 (2007).
- [3] A. W. Steiner, M. Prakash, J.M. Lattimer and P.J. Ellis, Phys. Rep. **411**, 325 (2005).
- [4] B.A. Li, C.M. Ko, and W. Bauer, Int. J. Mod. Phys. E**7**, 147 (1998).
- [5] *Isospin Physics in Heavy-Ion Collisions at Intermediate Energies*, Eds. B. A. Li and W. Uuo Schröder (Nova Science Publishers, Inc, New York, 2001).
- [6] V. Baran, M. Colonna, V. Greco and M. Di Toro, Phys. Rep. **410**, 335 (2005).
- [7] L. W. Chen, C. M. Ko, B.A. Li and G.C. Yong, Frontiers of Physics in China, **2**, 327 (2007).
- [8] B.A. Li, L.W. Chen, and C. M. Ko, Phys. Rep. **464**, 113 (2008).
- [9] J.S. Read, B.D. Lackey, B.J. Owen and J. L. Friedman, Phys. Rev. **D79**, 124032 (2009).

- [10] F. Ozel, G. Baym, and T. Guver, Phys. Rev. D **82**, 101301 (2010).
- [11] A.W. Steiner, J.M. Lattimer, and E.F. Brown, Astrophys. J. **722**, 33 (2010).
- [12] P. Danielewicz, R. Lacey, and W.G. Lynch, Science **298**, 1592 (2002).
- [13] C. Fuchs, Prog. Part. Nucl. Phys. **56**, 1 (2006).
- [14] Wolfgang Trautmann, talk at the XLVIII International Winter Meeting on Nuclear Physics, Bormio (Italy), <http://panda.physik.uni-giessen.de:8080/indico/conferenceDisplay.py?confId=16>
- [15] SEP project, <http://groups.nsl.msu.edu/hira/sep.htm>
- [16] Samurai collaboration, http://rarfaxp.riken.go.jp/RIBF-TAC05/10_SAMURAI.pdf
- [17] U. Garg, arXiv:1101.3125
- [18] A.W. Steiner, B.A. Li, Phys. Rev. C **72**, 041601 (R) (2005).
- [19] B.A. Li and A.W. Steiner, Phys. Lett. B **642**, 436 (2006).
- [20] Aaron Worley, Plamen G. Krastev and Bao-An Li, Astrophys. J. **685**, 390 (2008).
- [21] Plamen G. Krastev, Bao-An Li and Aaron Worley, Astrophysical J. **676**, 1170 (2008).
- [22] J. Xu, L. W. Chen, B. A. Li, and H. R. Ma, Astrophysics J. **697**, 1549 (2009).
- [23] W.G. Newton and B.A. Li, Phys. Rev. C **80**, 065809 (2009).
- [24] A.M. Lane, Nucl. Phys. **35**, 676 (1962).
- [25] K. A. Brueckner and J. Dabrowski, Phys. Rev. **134**, B722 (1964).
- [26] J. Dabrowski and P. Haensel, Phys. Lett. B **42**, 163 (1972); Phys. Rev. C **7**, 916 (1973); Can. J. Phys. **52**, 1768 (1974).
- [27] C. Xu, B.A. Li, and L.W. Chen, Phys. Rev. C **87**, 054607 (2010).
- [28] N.M. Hugenholtz, L. Van Hove, Physica **24**, 363 (1958).
- [29] C. Xu, B.A. Li, L.W. Chen, and C.M. Ko, arXiv:1004.4403, Nucl. Phys. A (2011) to be published.
- [30] E.N.E. van Dalen, C. Fuchs and A. Faessler, Phys. Rev. C **72**, 065803 (2005).
- [31] W. Zuo, L.G. Cao, B.A. Li, U. Lombardo, and C.W. Shen, Phys. Rev. C **72**, 014005 (2005).
- [32] L.W. Chen, C.M. Ko, and B.A. Li, Phys. Rev. C **72** (2005) 064606.
- [33] Z.H. Li, L.W. Chen, C.M. Ko, B.A. Li, and H.R. Ma, Phys. Rev. C **74** (2006) 044613.
- [34] V. Giordano, M. Colonna, M. Di Toro, V. Greco and J. Rizzo, Phys. Rev. C **81**, 044611 (2010).
- [35] M.A. Preston and R.K. Bhaduri, Structure of the Nucleus, Addison-Wesley, Reading, MA, 1975, p. 191-202.
- [36] C. Xu and B.A. Li, Phys. Rev. C **81**, 064612 (2010).
- [37] Hyun Kyu Lee, Byung-Yoon Park and Mannque Rho, Phys. Rev. C **83**, 025206 (2011).
- [38] H. Dong, T.T.S. Kuo and R. Machleidt, Phys. Rev. C **80**, 065803 (2009); arXiv:1101.1910
- [39] T. Frick, H. Muther, A. Rios, A. Polls and A. Ramos, Phys. Rev. C **71**, 014313 (2005).
- [40] W. D. Myers, W. J. Swiatecki, Nucl. Phys. A **81**, 1 (1966).
- [41] P. Möller *et al.*, Atom. Data Nucl. Data Tabl. **59**, 185 (1995).
- [42] K. Pomorski, J. Dudek, Phys. Rev. C **67**, 044316 (2003).
- [43] M. Liu, N. Wang, Z. Li, and F. Zhang, Phys. Rev. C **82**, 064306 (2010).
- [44] M.B. Tsang *et al.*, Phys. Rev. Lett. **92**, 062701 (2004).
- [45] M.B. Tsang *et al.*, Phys. Rev. Lett. **102**, 122701 (2009).
- [46] M.A. Famiano *et al.*, Phys. Rev. Lett. **97**, 052701 (2006).
- [47] L.W. Chen, C.M. Ko and B.A. Li, Phys. Rev. Lett. **94** (2005) 032701.
- [48] B.A. Li and L.W. Chen, Phys. Rev. C **72** (2005) 064611.
- [49] D. V. Shetty, S. J. Yennello, and G. A. Souliotis, Phys. Rev. C **76**, 024606 (2007).
- [50] P. Danielewicz and J. Lee, Nucl. Phys. A **818**, 36 (2009).
- [51] M. Centelles, X. Roca-Maza, X. Vinas, and M. Warda, Phys. Rev. Lett. **102**, 122502 (2009).
- [52] M. Warda, X. Vinas, X. Roca-Maza, and M. Centelles, Phys. Rev. C **80**, 024316 (2009).
- [53] L.W. Chen, C.M. Ko, B.A. Li, and J. Xu, Phys. Rev. C **82**, 024321 (2010).
- [54] A. Klimkiewicz *et al.*, Phys. Rev. C **76**, 051603(R) (2007).
- [55] A. Carbone *et al.*, Phys. Rev. C **81**, 041301 (R) (2010).
- [56] L.W. Chen, arXiv:1101.5217, Phys. Rev. C (2011) in press.
- [57] A.W. Steiner and A.L. Watts, Phys. Rev. Lett. **103**, 181101 (2009).
- [58] L. Trippa, G. Colò, and E. Vigezzi, Phys. Rev. C **77**, 061304(R) (2008).
- [59] L.G. Cao and Z.Y. Ma, Chin. Phys. Lett. **25**, 1625 (2008).
- [60] C.J. Horowitz, A. Schwenk, Nucl. Phys. A **776** (2006) 55.
- [61] S. K. Samaddar, J. N. De, X. Vinas and M. Centelles, Phys. Rev. C **80**, 035803 (2009).
- [62] S. Typel, G. Ropke, T. Klahn, D. Blaschke, H. H. Wolter, Phys. Rev. C **81**, 015803 (2010).
- [63] J. B. Natowitz *et al.*, Phys. Rev. Lett. **104**, 202501 (2010).
- [64] C.B. Das, S. DasGupta, C. Gale and Bao-An Li, Phys. Rev. C **67**, 034611 (2003).
- [65] L.W. Chen, C.M. Ko and B.A. Li, Phys. Rev. C **76**, 054316 (2007).
- [66] B.A. Li, Phys. Rev. Lett. **85** (2000) 4221.

- [67] B.A. Li, Phys. Rev. Lett. **88**, 192701 (2002); Nucl. Phys. **A708**, 365 (2002).
- [68] G.C. Yong, B.A. Li and L.W. Chen, Phys. Lett. **B650**, 344 (2007).
- [69] Z. G. Xiao, B. A. Li, L. W. Chen, G. C. Yong and M. Zhang, Phys. Rev. Lett. **102**, 062502 (2009).
- [70] J. Xu, C. M. Ko, and Y. Oh, Phys. Rev. C **81**, 024901 (2010).
- [71] Z.Q. Feng and G.M. Jin, Phys. Lett. **B683**, 140 (2010).
- [72] P. Russotto, P.Z. Wu, M. Zoric, M. Chartier, Y. Leifels, R.C. Lemmon, Q. Li, J. Lukasik, A. Pagano, P. Pawlowski and W. Trautmann, Physics Letters **B697**, 471 (2011).
- [73] M.D. Cozma, arXiv:1102.2728
- [74] C.J. Horowitz and J. Piekarewicz, Phys. Rev. Lett. **86**, 5647 (2001); Phys. Rev. C **64**, 062802 (2001).
- [75] M. Kutschera, Phys. Lett. **B340**, 1 (1994); Z. Phys. **A348**, 263 (1994); Acta Phys. Polon. **B29**, 25 (1998).
- [76] A. W. Steiner, Phys. Rev. C **74**, 045808 (2006).
- [77] S. Kubis, Phys. Rev. C **76**, 025801 (2007).
- [78] D.H. Wen, B.A. Li and L.W. Chen, Phys. Rev. Lett. **103**, 211102 (2009).
- [79] Steven Weinberg, page 303 in *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, John Wiley & Sons, Inc., 1972.
- [80] James B. Hartle, *Gravity*, Addison Wesley, 2003.
- [81] W.K. Lin, B.A. Li, J. Xu, C. M. Ko, and D.H. Wen, arXiv:1011.6073.
- [82] J. Xu, L.W. Chen, C.M. Ko, and B.A. Li, Phys. Rev. C **81**, 055803 (2010).
- [83] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D **9**, 12 (1974).
- [84] U. Heinz, P. R. Subramanian, H. Stocker and W. Greiner, Nucl. Phys. **12**, 1237 (1986).
- [85] N.K. Glendenning, Phys. Rev. D **46**, 1274 (1992).
- [86] N.K. Glendenning, Phys. Rep. **342**, 393 (2001).
- [87] M. Kutschera and J. Niemiec, Phys. Rev. C **62**, 025802 (2000).
- [88] S. Chandrasekhar and V. Ferrari, Proc. R. Soc. Lond. A **432**, 247 (1991); *ibid* **434**, 449 (1991).
- [89] P.G. Krastev, B.A. Li and A. Worley, Phys. Lett. **B668**, 1 (2008).
- [90] A. Worley, P.G. Krastev and B.A., Li, arXiv:0812.0408.
- [91] Plamen G. Krastev, Bao-An Li, and Aaron Worley, Phys. Lett. B **668**, 1 (2008).
- [92] D. H. Wen, B. A. Li and P. G. Krastev, Phys. Rev. C, **80**, 025801 (2009).
- [93] Plamen G. Krastev and Bao-An Li, Chapter 5 in *Pulsars: Theory, Categories and Applications*, Editor: Alexander D. Morozov, Nova Sciences Publishers Inc. (New York, 2010), ISBN: 978-1-61668-919-3.
- [94] The LIGO Collaboratiuon, <http://www.ligo.caltech.edu>
- [95] The LISA Collaboration, <http://www.lisa.aei-hannover.de>